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Radar Inverse Scattering Using Statistical Estimation of the Echo Phase-Front Derivatives

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FOREWORD

The research described in this report was performed in fiscal year 1986 as a part of an effort to apply robust statistical estimation techniques to radar inverse scattering. The goal in radar inverse scattering is to identify an unknown radar target by techniques that make fundamental use of the physics of electromagnetic scattering. Inverse scattering methods yield basic descriptors of targets, such as lengths, diameters, and radii of curvature. The hope is that these descriptors will permit target identification without the computer intensive signal template matching required by conventional target identification methods. This effort was supported by 6.1 funds from Naval Air Systems Command, Code 933; support for this work has continued in fiscal year 1987 under funding from the Office of Naval Research, Code 1247.

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(U) Many of the existing methods for asymptotic inverse scattering rely heavily on simplifying assumptions such as convex target structure and full-aspect data sets. These assumptions are rarely justified when applied to radar target identification. Investigated here is a statistical method appropriate to very complex targets consisting of many scattering centers, each scatterer with different morphological properties. Under the assumption that the data are collected from a very narrow range of "unknown" aspects and using only several closely spaced frequencies, we show that symmetric targets can be reconstructed by measuring the statistical properties of the echo phase-front spatial derivatives. A practical technique for extracting the required derivative information is discussed.

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INTRODUCTION

Many approaches to electromagnetic inverse scattering allow reconstruction of target shape only by assuming broadband data over a large, or even complete, set of aspects (References 1 and 2). Because of problems with nonuniqueness of solutions, this is often necessary. Unfortunately, however, such stringent requirements limit the applicability of the developed techniques. In radar-target-identification problems, for example, we typically are restricted to high frequencies and an unknown but very small range of aspects (aspects being defined by target orientation). Adding to these difficulties are typical complex target structures consisting of distributed (perhaps interacting) scatterers with varying reflectivities and associated intrinsic phase shifts. In fact, very few target characteristics can be determined with certainty; we must approach the problem statistically.

Surprisingly, statistical methods in theoretical inverse scattering have not found much favor in problems involving target-shape estimation. Moreover, many of the applications-oriented developments tend to be <u>ad hoc</u> in nature and rely heavily on large data bases and the special training of human operators. These methods are often based on the amplitude statistics of the scattering centers and frequently require extensive <u>a priori</u> information before they can be considered useful.

We develop a statistical inverse method that allows for shape reconstruction of such symmetric complex targets. These targets are assumed to consist of many independent scattering centers distributed within a finite region of space and are expected to "wobble" (with time) in a small but unknown manner. The data used in the reconstruction consist of phase derivatives of the kind that are currently used for phase-difference radar tracking. We proceed by first examining the statistics of these phase derivatives, then showing how they are related to target shape, and finally by developing a shape-reconstruction algorithm based on these results. Examples using synthetic data are displayed. We finish by describing a proposed method for measuring the required phase-derivative statistics with a practical antenna array.

STATISTICS OF THE PHASE DERIVATIVES

A statistical model based on cross sections can hardly be expected to be of much utility in shape determination for complex targets, because scattering amplitude depends in a sensitive and complicated way on local scattering center structure and morphology. However, it has long been realized that a wealth of target-shape information is contained in the phase of the asymptotic scattered field. The trouble is, of course, that this phase information is very difficult to obtain with any accuracy over a set of aspects. On the other hand, phase-derivative information can be approximated to good accuracy by measuring the phase differences between closely spaced sensors. In radar applications, for example, the method is known as phase monopulse and has been extensively studied in its relationship to tracking problems (Reference 3). Because of practical considerations, such as noisy measurements and inaccurate aspect information, the first derivatives are determined by these kinds of "point" measurements using two receiving antennas. In general, the J^{th} derivative would require an array of J + 1 antennas.

If $F \equiv Ee^{i\phi}$ denotes the scattered field, then the phase is just

$$\phi = \arctan\left(-i\,\frac{F - F^*}{F + F^*}\right)$$

so that

$$\frac{\partial\phi}{\partial\eta} = i\,\frac{F\,\partial F^*/\partial\eta - F^*\,\partial F/\partial\eta}{2FF^*} \tag{1}$$

Consider a situation in which the scattering centers associated with a complex extended body are excited by an incident plane wave originating from an observer located at a distance R from the body. (The coordinate system is fixed within the scatterer.) The high-frequency time harmonic far-field scattered from a general target can be written in the form (Reference 4)

$$F(R) = \frac{E_o e^{i2kR}}{2\pi R} \sum_{j=1}^{N} A_j e^{i(2k\hat{R} - r_j + \phi_j)}$$

where the scatterer is taken to be composed of N (possibly infinite) scattering centers with associated (real) scattering strengths A_j, positions r_j, and intrinsic phase shifts ϕ_j. E_o is the (constant) magnitude of the incident field, R is a unit vector directed away from the observer, $k = 2\pi/\lambda$ is the wave number, and $2(R \cdot r_j + R)$ is the (two-way) wave-travel distance from the observer to the j^{th} scatterer. (We have suppressed the $e^{i\omega t}$ time dependence.)

In a standard cartesian coordinate system in which R lies along the z direction and r_j has components (u_j, v_j, w_j), define

$$E_{m_1,m_2,m_3}\, e^{i\phi_{m_1,m_2,m_3}} \equiv \frac{E_o e^{i2kR}}{2\pi R} \sum_{j=1}^{N} \left(\frac{2k}{R}\right)^{m_1+m_2+m_3}$$

$$\times A_j u_j^{m_1} v_j^{m_2} w_j^{m_3}\, e^{i(2k\hat{R} - r_j + \phi_j)}$$

where the m_i are integers. In terms of these definitions, the various phase derivatives can be determined from Equation 1. In the high-frequency limit, we have for example:

$$\chi_{100} \equiv \frac{\partial \phi}{\partial x} = \left(\frac{E_{100}}{E}\right)\cos(\phi-\phi_{100})$$

$$\chi_{200} \equiv \frac{\partial^2 \phi}{\partial x^2} = \left(\frac{E_{200}}{E}\right)\sin(\phi-\phi_{200}) - \left(\frac{\partial \phi}{\partial x}\right)\left(\frac{E_{100}}{E}\right)\sin(\phi-\phi_{100})$$

$$\chi_{101} \equiv \frac{\partial^2 \phi}{\partial x\, \partial k} = \left(\frac{E_{101}}{E}\right)\sin(\phi-\phi_{101}) - \left(\frac{\partial \phi}{\partial k}\right)\left(\frac{E_{100}}{E}\right)\sin(\phi-\phi_{100}) \qquad (2)$$

and so forth. (We assume that only the central element of the antenna array is used for estimating wave-number derivatives.)

To obtain the statistics of the $\chi_{\ell mn}$, we make the following assumptions. We take $\{A_j\}$ to be a set of indecendent random variables, each A_j having the same statistical frequency distribution

$f_A(A_j)$. For our complex physical scatterer, we also take $\{\phi_j\}$ to be a set of independent random variables symmetrically distributed on $(-\pi,\pi)$ with distribution $f_\phi(\phi_i)$. This assumption is not only appropriate, it greatly simplifies the following calculation. Moreover, at high frequencies (for which λ is much less than a "typical" target dimension), it clearly will not alter significantly the results that would be obtained by assuming the target to be composed of noninteracting, convex, and perfectly conducting scattering centers. Finally, we suppose that the aspect angle through which the target rotates is sufficiently small so that the statistics are stationary and so that each $A_j(\hat{R})$ and $\phi_j(\hat{R})$ remain essentially fixed.

We begin by examining the statistics of

$$\xi_{\ell mn} = \frac{E_{\ell mn}}{E} \cos(\phi - \phi_{\ell mn}) \tag{3}$$

Define

$$s \equiv \textstyle\sum_j A_j \cos(\beta_j), \quad t \equiv \sum_j A_j \sin(\beta_j)$$

$$u \equiv \textstyle\sum_j A_j \alpha_j \cos(\beta_j), \text{ and } v \equiv \sum_j A_j \alpha_j \sin(\beta_j)$$

where $\alpha_j \equiv (2ku_j/R)^\ell (2kv_j/R)^m (2w_j)^n$, $\beta_j \equiv 2kR - r_j + \phi_j$, and the sums are taken to range from 1 to N. (Suppressing the (ℓ,m,n) dependence will not cause confusion in the following.) Then we can write

$$\xi_{\ell mn} = \frac{su + tv}{s^2 + t^2}$$

Under the stated assumptions, we can show

$$\langle s\rangle = \langle t\rangle = \langle u\rangle = \langle v\rangle = 0$$

$$\sigma_s^2 \equiv \langle s^2\rangle - \langle s\rangle^2 = \sigma_t^2, \text{ and } \sigma_u^2 = \sigma_v^2$$

It is easy to see (by calculating cross correlations) that s, t, u, and v are statistically independent. Since each of these terms can be written as a sum $S = \Sigma\zeta_j$, where ζ_j are identically distributed with nonvanishing finite variances, they each satisfy the Lindberg conditions (Reference 5) and so have asymptotically normal frequency distributions.

It is a straightforward calculation to show that both E and $E_{\ell mn}$ of Equation 3 are independent Rayleigh-distributed random variables. The independence of ϕ and $\phi_{\ell mn}$ follows from the independence of the normally distributed complex variables $E\exp(i\phi)$ and $E_{\ell mn}\exp(i\phi_{\ell mn})$, so that the product $E_{\ell mn}\cos(\phi - \phi_{\ell mn})$ is normally distributed and independent of E. Finally, we can conclude that the statistics of $\xi_{\ell mn}$, as the quotient of normal and Rayleigh-distributed independent random variables, can be expressed as a Student's "t" distribution:

$$g_{\ell mn}(\xi) = \frac{1}{2}\frac{\Omega^2_{\ell mn}}{(\Omega^2_{\ell mn} + \xi^2)^{3/2}}, \tag{4}$$

where

$$\Omega^2_{\ell mn} \equiv \frac{\sigma_u^2}{\sigma_s^2} = \lim_{N\to\infty}\frac{\sum_j\iint A_j^2\alpha_j^2\cos^2(\beta_j)f_\phi(\phi_j)f_A(A_j)d\phi_j dA_j}{\sum_j\iint A_j^2\cos^2(\beta_j)f_\phi(\phi_j)f_A(A_j)d\phi_j dA_j}$$

To write this in a more tractable form, observe that a scatterer of strength $M \times A$, located at r_j, has scattered intensity equivalent to M (in-phase) scatterers of strength A at the same location. Define a weighted density function by

$$A^2\rho(r_j)\Delta V_j \equiv \int A_j^2 f_A(A_j)dA_j$$

where A is a constant and ΔV_j is an incremental volume about r_j. Substitution yields

$$\Omega^2_{\ell mn} = \lim_{N\to\infty} \frac{A^2 \sum_j \alpha_j^2 \rho(r_j) \Delta V_j}{A^2 \sum_j \rho(r_j) \Delta V_j} = \int_{-\infty}^{\infty} \alpha^2 \rho(r) d^3r$$

(See Reference 5 for the details of these calculations.)

Of course, this result gives the frequency distribution of the first derivatives of the phase. The frequency distributions of the higher derivatives follow from this result. For example, from Equation 2 we see that the second derivative of ϕ with respect to x can be treated as a function of three independent Student's "t" distributed random variables:

$$z = s_1 - s_2 s_3$$

We can compute the statistical distribution of z in the usual way as

$$f_z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{200}(z + r)\, g_{100}(\frac{r}{t})\, g_{100}(t) dt dr$$

where the $g_{\ell mn}$ are defined by Equation 4. Making the substitution $\eta = t^2/r$, the integral over t can be seen to be of the tabulated variety (Reference 6). After some algebra, we are left with

$$f_z = \frac{\Omega^2{}_{200}\, B(3/2,3/2)}{2[(\Omega^2{}_{100}-z)^2 + \Omega^2{}_{200}]^{3/2}} \times$$

$$\int_{-1}^{1} \frac{(1 - y)^3}{(1 + y)} \frac{F(1/2,3/2;2;y^2)}{(y^2 + 2by + a)^{3/2}} dy$$

where

$$a \equiv \frac{(\Omega^2{}_{100} + z)^2 + \Omega^2{}_{200}}{(\Omega^2{}_{100} - z)^2 + \Omega^2{}_{200}}$$

$$b \equiv \frac{(\Omega^2{}_{100} - z)(\Omega^2{}_{100} + z) - \Omega^2{}_{200}}{(\Omega^2{}_{100} - z)^2 + \Omega^2{}_{200}}$$

$$y \equiv \frac{r - \Omega^2{}_{100}}{r + \Omega^2{}_{100}}$$

B is a beta function, and F is a hypergeometric function. We shall satisfy ourselves with an approximation to this last integral.

Observing that the term $(1 - y)^3F(1/2, 3/2; 2; y^2)/(1 + y)$ is singular at $y = -1$ and decays very rapidly to 0 thereafter, we take

$$f_z \approx C \int_{-1}^{1} \delta(y + 1)(y^2 + 2by + a)^{-3/2}\, dy$$

where C is a normalization constant and δ is a delta function. Our final result follows easily from this last integral.

We have shown that the distributions of the phase derivatives $\chi_{\ell mn}$ (Equation 2) obey

$$f_{100}(\chi) = \frac{1}{2}\, \Omega_{100}{}^2(\Omega_{100}{}^2 + \chi^2)^{-3/2}$$

$$f_{200}(\chi) \approx \frac{1}{2}\, \Omega_{200}{}^2(\Omega_{200}{}^2 + \chi^2)^{-3/2}$$

$$f_{101}(\chi) \approx \frac{1}{2}\, \Omega_{101}{}^2(\Omega_{101}{}^2 + \chi^2)^{-3/2} \qquad (5)$$

etc. ..., where

$$\Omega_{100}{}^2 = \left(\frac{2k}{R}\right)^2 \int u^2 \rho(r) d^3r$$

$$\Omega_{200}{}^2 = \left(\frac{2k}{R}\right)^4 \int u^4 \rho(r) d^3r$$

$$\Omega_{101}{}^2 = 2^2 \left(\frac{2k}{R}\right)^2 \int u^2 w^2 \rho(r) d^3r$$

The distributions of the higher derivatives follow from similar considerations.

In the next section, we shall show how the $\Omega_{\ell mn}$ can be related to target structure.

TARGET-SHAPE DESCRIPTION

In the following, we take as our target-shape descriptor the scattering center density function $\rho(r)$. Associated with this is a characteristic function defined by

$$\theta(\lambda) = \int_{-\infty}^{\infty} \rho(r) e^{i\lambda - r} \quad d^3r \tag{6}$$

Expanding $\theta(\lambda)$ by Maclaurin's formula yields

$$\theta(\lambda) = 1 + \sum_{n=1}^{\nu} i^n \sum_{m_1, m_2, m_3 = n} \mu_{m_1, m_2, m_3} \frac{\lambda_1{}^{m_1} \lambda_2{}^{m_2} \lambda_3{}^{m_3}}{m_1! m_2! m_3!} + R\nu \tag{7}$$

where

$$\mu_{m_1, m_2, m_3} \equiv i^{-n} \left[\frac{\partial^n \theta(\lambda)}{\partial \lambda_1{}^{m_1} \partial \lambda_2{}^{m_2} \partial \lambda_3{}^{m_3}}\right] \lambda = 0$$

and R_ν is a remainder term: $\lim_{\nu\to\infty} R_\nu = 0$. Substitution of Equation 6 into this last definition yields

$$\mu_{m_1,m_2,m_3} = \int_{-\infty}^{\infty} u^{m_1} v^{m_2} w^{m_3} \rho(r) d^3r$$

(For example, the moments of $\rho(r)$ determine $\rho(r)$ by inversion of the transform Equation 6.)

Targets symmetric across the coordinate planes will have vanishing "odd" moments. Such targets can, therefore, be reconstructed from a knowledge of their "even" moments only. We have seen that the statistics of the phase derivatives depend in a simple way on these even moments, and it is a straightforward matter to extract this information from a phase-derivative data set.

ESTIMATION OF $\Omega^2_{\ell mn}$

Given a collection of M-phase-derivative data points gathered from the target modeled in the previous discussion, we seek to estimate $\Omega_{\ell mn}{}^2 = (2k/R)^{2(1+m)2n}\ \mu_{2\ell,2m,2n}$. For a complex target, it is known that $\chi_{\ell mn}$ is a rapidly varying function of aspect. Therefore, we are often in the position of looking at a widely varying data set from a target, which itself varies only slightly in presented aspect. For this situation, we can determine the value of $\Omega_{\ell mn}$ that best matches these data to the Student's "t" distribution law that is expected.

Assuming the statistics of Equation 5, the maximum-likelihood estimator for $\Omega_{\ell mn}$ will be the one that maximizes the function (Equation 7)

$$\Lambda(\Omega_{\ell mn}) = \prod_{i=1}^{M} \frac{\Omega^2_{\ell mn}/2}{(\Omega^2_{\ell mn} + \chi_i^2)^{3/2}}$$

where the product is taken over all the M elements of the data set $\{\chi_i\}$. Here, we choose instead the value of $\Omega_{\ell mn}$ that maximizes the logarithm of $\Lambda(\Omega_{\ell mn})$ (which will also maximize $\Omega_{\ell mn}$), for obvious reasons.

Differentiating $\ell n\Lambda(\Omega_{\ell mn})$ with respect to $\Omega_{\ell mn}$ and setting the result to zero yields the relation

$$\frac{1}{2}\sum_{i=1}^{M}\frac{\Omega_{\ell mn}^{2}}{(\Omega_{\ell mn}^{2}+\chi_{i}^{2})^{3/2}}=\frac{M}{3}$$

Solving this equation is tantamount to finding the real roots of a polynomial of degree 2M, but since it is the sum of functions of the form

$$\frac{\Omega^{2}/2}{\Omega^{2}+\chi_{i}^{2}}-\frac{1}{3}$$

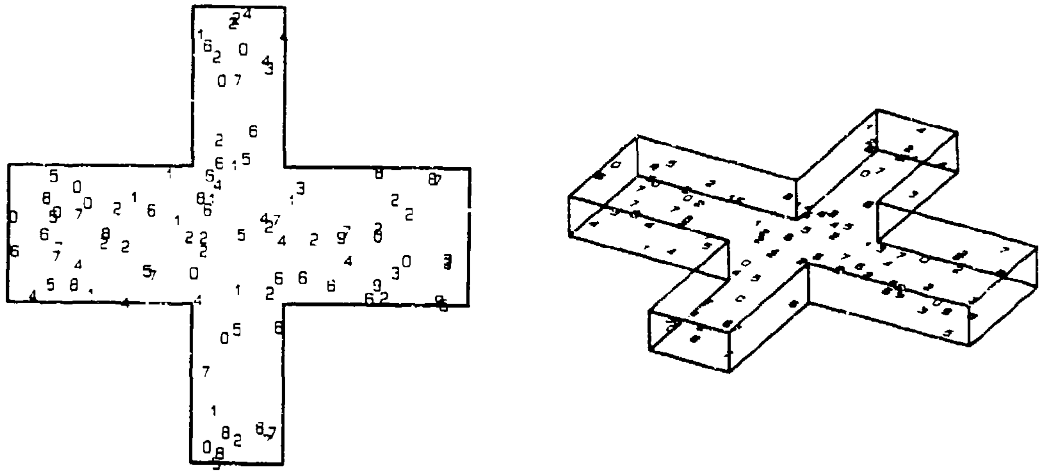
then there will be only two real zeros, which will be located symmetrically about the origin, and the magnitude of either zero is the solution. Thus, we can use an iterative scheme to solve for $\Omega_{\ell mn}$.

SAMPLE RESULTS

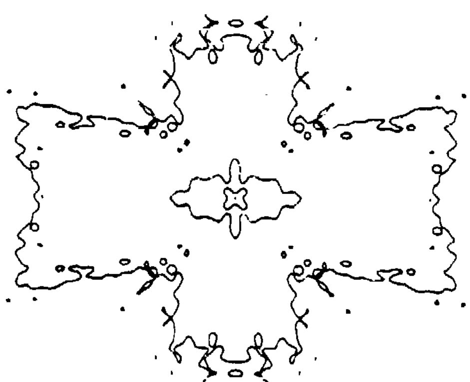
Targets of the kind that we have been modeling allow for the easy construction of simulated data sets. This is because we are not trying to locate the actual scattering centers associated with a specific target. Instead, we are free to create structured collections of scatterers and investigate the ability of the devised algorithm to accurately reconstruct them from the calculated phase-derivative data. To accomplish such a verification, we developed a computer simulation that (1) creates a statistical target model, (2) determines the relevant phase-derivative information over a small range of random aspects, (3) estimates the corresponding moments of the scattering center distribution function, and (4) reconstructs the "target" by inverting Equation 6.

Each of the targets displayed in Figures 1 through 3 consisted of 100 scattering centers randomly located within a plane-symmetric support "shape." Each scattering center was randomly assigned a scattering strength and associated phase shift. Data sets consisting of the first, second, and third derivatives of the scattered field phase were constructed by "viewing" the target from different aspects. We have chosen these aspects to lie within a small "cone" of directions

(≈ 3 × 3 degrees) centered on one of the multiple symmetry directions of the target.

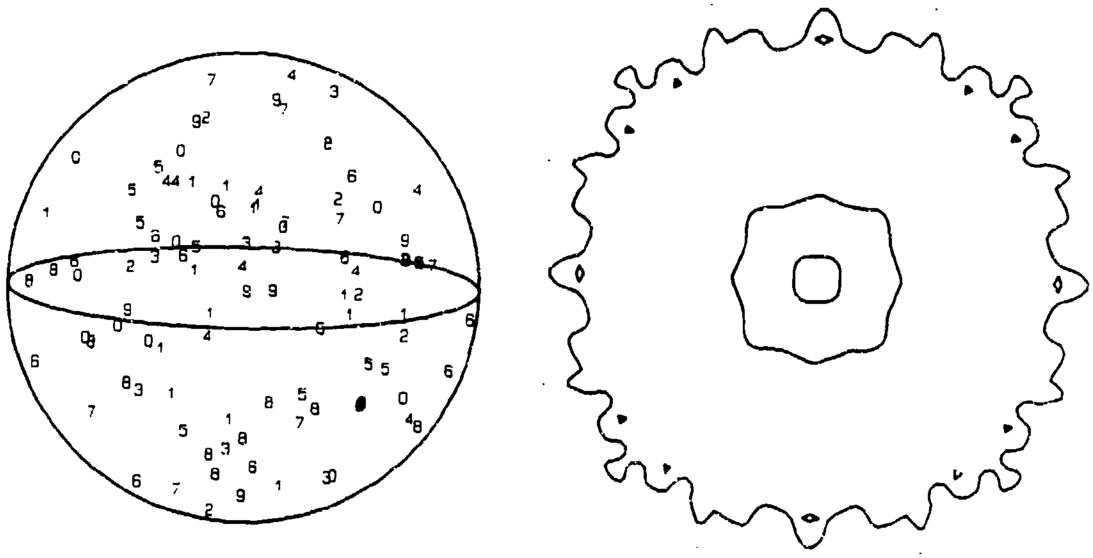


(a)



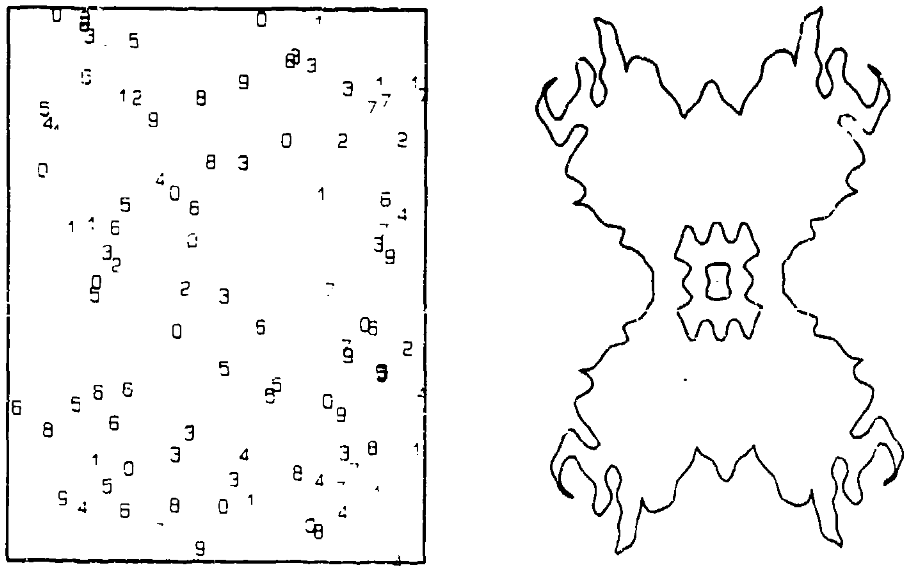
(b)

FIGURE 1. Synthetic Target (a) and Its Reconstructed Contour "Image" (b). The target consisted of 100 scattering centers randomly placed inside the displayed support with random scattering strengths (shown in the figure) and random local phase shifts. Target dimensions are 10 × 10 × 1; modeled wavelength is $\lambda = 10^{-2}$; and range is $R = 10^{3}$. The data consisted of the first three phase derivatives "collected" randomly from within a 3- × 3-degree range of aspects.



(a) (b)

FIGURE 2. Synthetic 100-Point Spherical Target (a) of Radius 10 and Its Reconstructed Image (b). All other parameters are as in Figure 1.



(a) (b)

FIGURE 3. Rectangular Plate Target (a) and the Image Reconstructed From Its Synthetically Determined Scattering Data. The support dimensions of the target are 15 × 20 × 1; all other parameters are as in Figure 1.

The target images were recaptured by using Equation 7 to load a (64 × 64 × 64) array and then using a three-dimensional fast-Fourier transform algorithm to invert Equation 6. Finally, the recaptured density function was collapsed along an axis to facilitate the plotting routine; thus the figures really represent an average density along this axis.

MEASUREMENTS OF THE PHASE-FRONT DERIVATIVES

For implementation in a practical radar, the technique presented in this report requires the measurement of the first three or so spatial derivatives of the echo phase front. This section discusses a technique for measuring these derivatives.

To measure the phase-front derivatives, the phase front is sampled and the finite difference approximation to the derivatives formed. We assume for this discussion that the first three derivatives are desired. Measurement of three derivatives requires that the phase front be sampled by four antennas, as shown in Figure 4, where for now it has been assumed that the axis of the array has been aligned normal

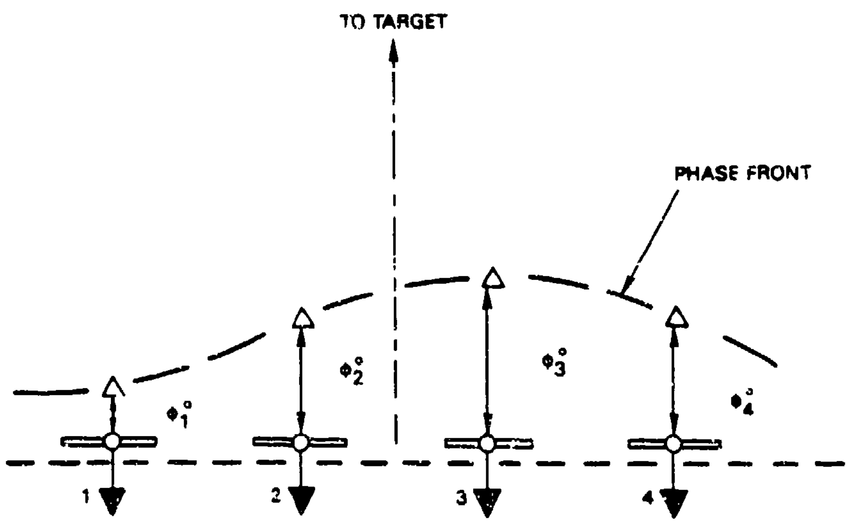


FIGURE 4. Four-Element Antenna Array With Impinging Wave. A sample locus of constant phase is shown. The ϕ_i represent the electrical distance $2\pi z_i/\lambda$ to the constant phase locus, where z_i is the physical distance.

to the direction to the target. That is, the tracking-angle error is zero. Denoting the phase at each antenna as $\phi_i{}^o$ ($i = 1...4$), the finite difference approximations to the first three differentials are given by

$$\Delta\phi^{(1)} = \phi_3^o - \phi_2^o \tag{8}$$

$$\Delta\phi^{(2)} = \phi_4^o - 2\phi_3^o + \phi_2^o \tag{9}$$

and

$$\Delta\phi^{(3)} = \phi_4^o - 3\phi_3^o + 3\phi_2^o - \phi_1^o \tag{10}$$

These differentials can be measured by the scheme shown in Figure 5. The phase discriminators produce an output that is the radio frequency (RF) phase difference between the two adjacent antenna elements. The path lengths from antenna elements to discriminators are constructed to be equal. The arrangement of difference amplifiers (whose bandwidth need only accommodate the rate of change of the RF phase, not the RF itself) forms Equations 8 through 10.

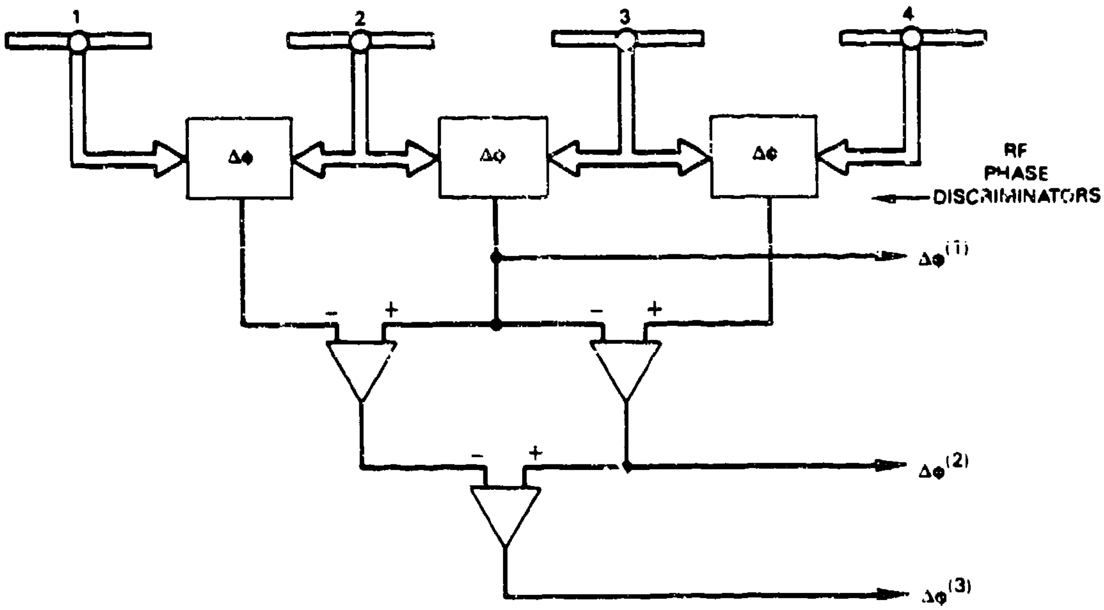


FIGURE 5. Interconnection of Four Antenna Elements to Extract the First Three Phase-Front Derivatives.

In general, some amount of tracking error will always exist (Figure 6), in which case (in the far field) the RF phase at each antenna is given by

$$\phi_1 = \phi_1^o$$

$$\phi_2 = \phi_2^o - \beta d \sin \theta$$

$$\phi_3 = \phi_3^o - 2\beta d \sin \theta$$

$$\phi_4 = \phi_4^o - 3\beta d \sin \theta$$

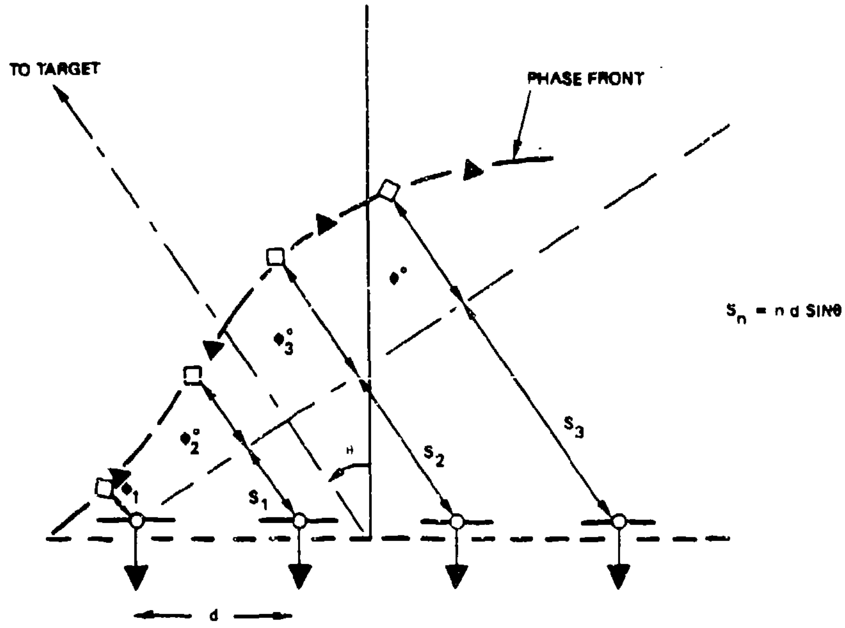


FIGURE 6. The Phase Front of Figure 4 Incident at an Angle θ. The squares present the new phase-front sampling points; the triangles are the sampling points in Figure 4.

The zero reference for the phase is taken arbitrarily at element 1. Note that for an identical wave front, different values of the ϕ_i^o are measured, because rotation of the wave front through an angle θ has the effect of decreasing the spacing between the antennas. This causes the wave front to be sampled at a different set of points. The ϕ_i^o, however, still represent the required phase values for forming the derivatives. Forming Equations 8 through 10, Equation 8 becomes

$$\Delta\phi^{(1)} = \phi_3^o - \phi_2^o - \beta d \sin \theta \qquad (11)$$

and Equations 9 and 10 are unchanged. Hence, a biased estimate of $\Delta\phi^{(1)}$ results, but $\Delta\phi^{(2)}$ and $\Delta\phi^{(3)}$ are unbiased. However, forcing Equations 8 through 10 to have zero mean (necessary conditions for this technique) will, for constant θ, remove the bias term in Equation 11. Clearly, problems may arise if the tracking loop allows fluctuations in θ that approach the rate of change of the phase differentials, thus causing θ to vary during the measurement interval. The tracking loop must be designed to prevent this in an application of the technique described in this report.

CONCLUSION

We have developed a statistical high-frequency inverse scattering method that allows the reconstruction of very complex targets from extemely limited data. These data consist of various phase derivatives collected over a small range of aspects and frequencies. Significantly, we do not require that this aspect dependence be known. We have assumed the target to be composed of many individual scattering centers, each with potentially different morphological properties; however, our result does not require that any specific assumptions be made about the statistics of the local scatterer strengths.

Because we have used the classical central-limit theorem to obtain the relevant statistics (and from these the even moments of the scattering center distribution functions), the technique allows only the reconstruction of symmetric targets.

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